PROCEDURE FOR DIGITAL IMAGE RESTORATION

CROSS-REFERENCE TO RELATED APPLICATION

This is a continuation-in-part of U.S. patent application Ser. No. 07/991,105, filed Dec. 16, 1992, now U.S. Pat. No. 5,414,782.

FIELD OF THE INVENTION

The present invention relates to image processing, and, in particular, to image restoration.

BACKGROUND OF THE INVENTION

Comprehensive coverage of prior art in the field of image restoration relevant to the system and method may be found in H. C. Andrews and B. R. Hunt, *Digital Image Restoration,* Prentice-Hall Signal Processing Series, Prentice-Hall, Inc., Englewood Cliffs, N.J. (1977); W. K. Pratt, *Digital Image Processing,* 2nd Edition, John Wiley and Sons, NY (1988); H. Stark, *Image Recovery, Theory and Application,* Academic Press, Inc., Harcourt Brace Jovanovich, Publishers, New York (1987); R. C. Gonzalez and P. Wintz, *Digital Image Processing,* 2nd Edition, Addison-Wesley Publishing Company, Inc., Advanced Book Program, Reading, Mass. (1987); and R. L. Lagendijk and J. Biemond, *Iterative Identification and Restoration of Images,* Kluwer International Series in Engineering and Computer Science, Kluwer Academic Publishers, Boston, Mass. (1991). For space-invariant blurs, digital image restoration is associated with the solution of two dimensional ill-posed convolution integral equations of the form

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}p(x-u,y-v)f(u,v)dudv, \qquad \text{Equation (1)}$$

where g(x,y) is the degraded image, f(x,y) is the desired ideal image, and p(x,y) is the known point spread function of the imaging system. The point spread function acts to blur or smooth out the ideal image, making it impossible to distinguish fine details in the recorded image g(x,y). Separately, g(x,y) is further contaminated by measurement noise. Thus:

$$g(x,y)=g_e(x,y)+n(x,y), \qquad \text{Equation (2)}$$

where $g_e(x,y)$ is the blurred image that would have been recorded in the absence of noise, and n(x,y) represents the cumulative effects of all noise processes affecting final acquisition of the digitized array g(x,y). This includes the case of multiplicative noise, where n(x,y) is a function of f(x,y). The noise component n(x,y) is unknown but may be presumed small. Likewise, $g_e(x,y)$ is unknown. The type and intensity of the blurring caused by p, together with the magnitude of the noise in g, ultimately limit the quality of the restoration that can be achieved. It is convenient to write Equation (1) in operation notation as:

$$Pf=g \qquad \text{Equation (3)}$$

where P is the integral operation in $L^2(R^2)$ with kernel p(x−u,y−v).

The two dimensional Fourier transform plays a major role in the subsequent analysis. For a function a(x,y) of the space

variables x,y, the Fourier transform $\hat{a}(\xi,\eta)$ may be expressed as:

$$\hat{a}(\xi,\eta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}a(x,y)e^{-2\pi i(\xi x+\eta y)}dxdy. \qquad \text{Equation (4)}$$

The Fourier transform of the point spread function is called the optical transfer function and is an important tool in image analysis. The convolution theorem states that the transform of a convolution is the product of the individual transforms. The Parseval theorem states that the L^2 scalar product of two functions in the space variables x,y is equal to their scalar product in the transform variables ξ,η. These and other results from Fourier analysis in L^2 are well known in the art. For convolution equations such as Equation (1), it is advantageous to perform the analysis and computations in the Fourier domain. After image processing, the inverse Fourier transform is used to return to the space variables x,y.

The class G of point spread functions, described by Equation (5A) below, plays a key role in several civilian and military applications, including biomedical imaging; night vision systems; undersea imaging; imaging through the atmosphere; remote sensing; high definition television; and industrial applications. Consider first the class of point spread functions p(x,y) described as follows in the Fourier transform domain:

$$\hat{p}(\xi,\eta)=e^{-\lambda(\xi^2+\eta^2)^\beta},\ \lambda>0,\ 0<\beta\leqq 1. \qquad \text{Equation (5)}$$

The case $\beta=1$ corresponds to a Gaussian point spread function. This case occurs in quite diverse applications, including undersea imaging (see H. T. Yura, 'Imaging in Clear Ocean Water,' *Applied Optics,* Vol. 12 (1973), pp. 1061–1066); low light-level electro-optical detection (see R. Weber, 'The Ground-Based Electro-Optical Detection of Deep-Space Satellites,' *Applications of Electronic Imaging Systems,* Proceedings of the Society of Photo-Optical Instrumentation Engineers, Vol. 143, R. E. Franseen and D. K. Schroder, Eds. (1978), pp. 59–69); nuclear medicine gamma camera scintigrams (see S. Webbet al., 'Constrained Deconvolution of SPECT Liver Tomograms by Direct Digital Image Restoration,' *Medical Physics,* Vol. 12 (1985), pp. 53–58; U. Raff et al., 'Improvement of Lesion Detection in Scintigraphic Images by SVD Techniques for Resolution Recovery,' *IEEE Transactions on Medical Imaging,* Vol. MI-5 (1986), pp. 35–44; B. C. Penney et al., 'Constrained Least Squares Restoration of Nuclear Medicine Images; Selecting the Coarseness Function,' *Medical Physics,* Vol. 14 (1987), pp. 849–859; B. C. Penney et al., 'Relative Importance of the Error Sources in Wiener Restoration of Scintigrams,' *IEEE Transactions on Medical Imaging,* Vol. 9 (1990), pp. 60–70; K. S. Pentlow et al., 'Quantitative Imaging of 1–124 Using Positron Emission Tomography with Applications to Radioimmunodiagnosis and Radioimmunotheray,' *Medical Physics,* Vol. 18 (1991), pp. 357–366); magnetic resonance imaging (see S. M. Mohapatra et al., 'Transfer Function Measurement and Analysis for a Magnetic Resonance Imager,' *Medical Physics,* Vol. 18 (1991), pp. 1141–1144); and computed tomography scanners (see E. L. Nickoloff and R. Riley, 'A Simplified Approach for Modulation Transfer Function Determinations in Computed Tomography,' *Medical Physics,* Vol. 12 (1985), pp. 437–442).

The case $\beta=5/6$ describes blurring caused by atmospheric turbulence under long time exposure. This optical transfer function is important in post processing of degraded images obtained in airborne reconnaissance, in remote sensing from